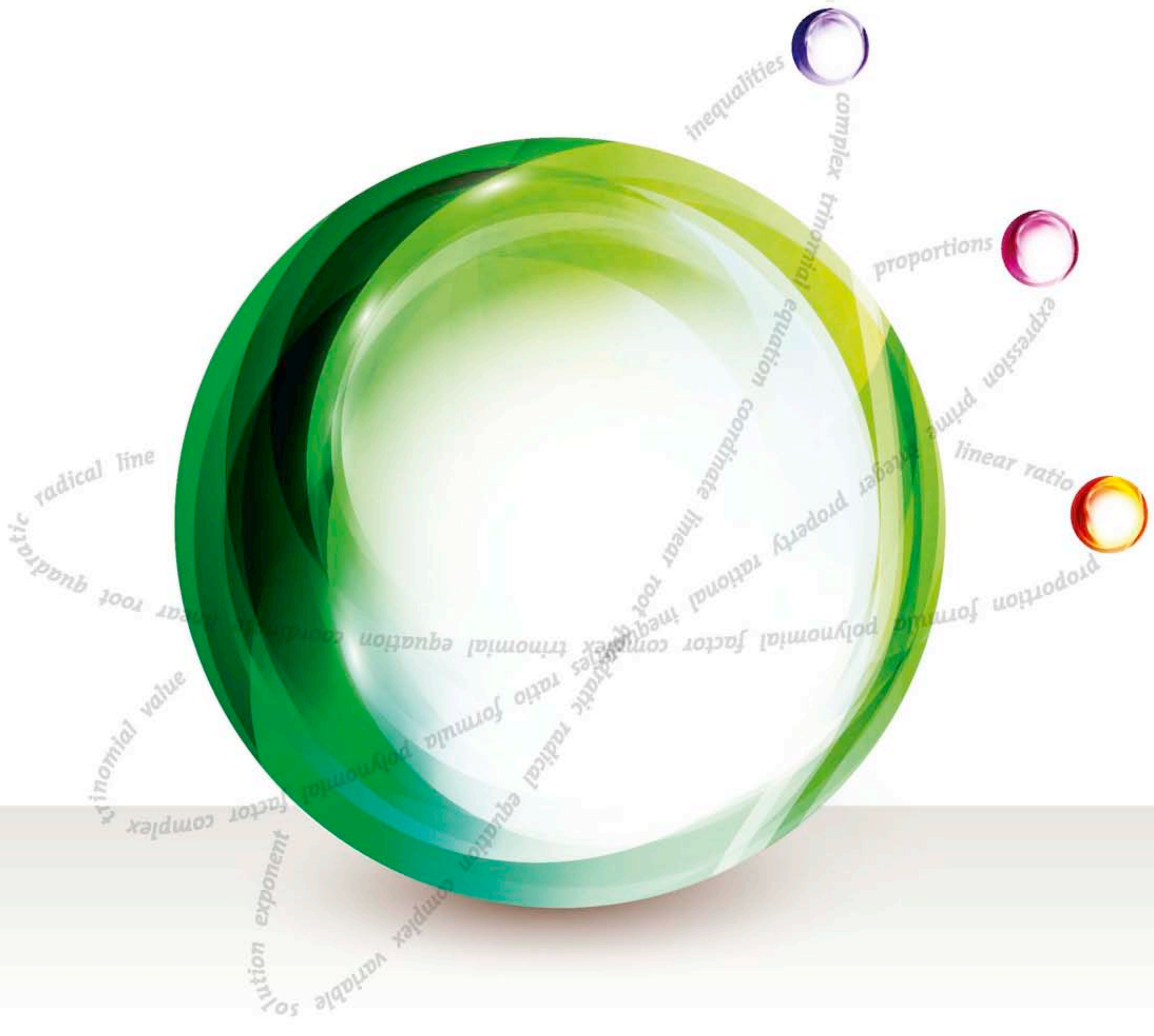
Intermediate Algebra 10e



Kaufmann / Schwitters

Symbols

$=$	Is equal to
$\neq$	Is not equal to
$\approx$	Is approximately equal to
$>$	Is greater than
$\geq$	Is greater than or equal to
$<$	Is less than
$\leq$	Is less than or equal to
$a < x < b$	a is less than x and x is less than b
$0.\overline{34}$	The repeating decimal 0.343434 . . .
LCD	Least common denominator
$\{a, b\}$	The set whose elements are a and b
$\{x \mid x \geq 2\}$	The set of all x such that x is greater than or equal to 2
$\varnothing$	Null set
$a \in B$	a is an element of set B
$a \notin B$	a is not an element of set B
$A \subseteq B$	Set A is a subset of set B
$A \nsubseteq B$	Set A is not a subset of set B
$A \cap B$	Set intersection
$A \cup B$	Set union
$\lvert x \rvert$	The absolute value of x
b^n	nth power of b
$\sqrt[n]{a}$	nth root of a
$\sqrt{a}$	Principal square root of a
i	Imaginary unit
$a + bi$	Complex number
$\pm$	Plus or minus
(a, b)	Ordered pair: first component is a and second component is b
f, g, h, etc.	Names of functions
$f(x)$	Functional value at x
$f \circ g$	The composition of functions f and g
f^{-1}	The inverse of the function f
$\log_b x$	Logarithm, to the base b, of x
$\ln x$	Natural logarithm (base e)
$\log x$	Common logarithm (base 10)
$n!$	n factorial

area A
perimeter P
length l

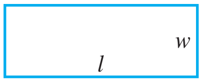
width w
surface area S
altitude (height) h

base b
circumference C
radius r

volume V
area of base B
slant height s

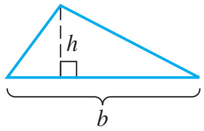
Rectangle

$A = lw \qquad P = 2l + 2w$



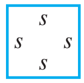
Triangle

$A = \frac{1}{2}bh$



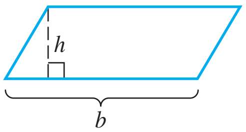
Square

$A = s^2 \qquad P = 4s$



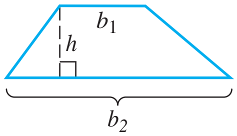
Parallelogram

$A = bh$



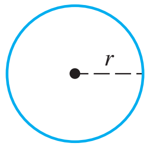
Trapezoid

$A = \frac{1}{2}h(b_1 + b_2)$

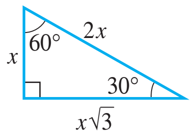


Circle

$A = \pi r^2 \qquad C = 2\pi r$

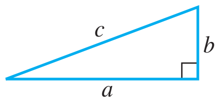


30°–60° Right Triangle

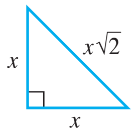


Right Triangle

$a^2 + b^2 = c^2$

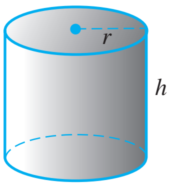


Isosceles Right Triangle



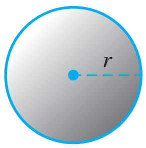
Right Circular Cylinder

$V = \pi r^2 h \qquad S = 2\pi r^2 + 2\pi rh$



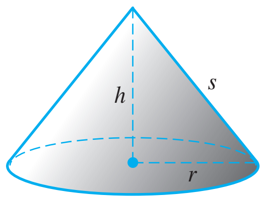
Sphere

$S = 4\pi r^2 \qquad V = \frac{4}{3}\pi r^3$



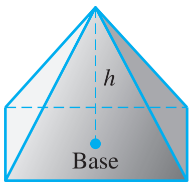
Right Circular Cone

$V = \frac{1}{3}\pi r^2 h \qquad S = \pi r^2 + \pi rs$



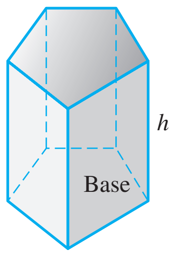
Pyramid

$V = \frac{1}{3}Bh$



Prism

$V = Bh$



TENTH EDITION

Intermediate Algebra

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Intermediate Algebra

Jerome E. Kaufmann
Karen L. Schwitters
Seminole State College of Florida

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Preface

When preparing *Intermediate Algebra, Tenth Edition*, we wanted to preserve the features that made the previous editions successful and, at the same time, incorporate improvements suggested by reviewers.

This text was written for college students who need an algebra course that bridges the gap between elementary algebra and the more advanced courses in precalculus mathematics. It covers topics that are usually classified as intermediate algebra topics.

The basic concepts of intermediate algebra are presented in this text in a simple, straightforward way. Algebraic ideas are developed in a logical sequence and in an easy-to-read manner without excessive formalism. Concepts are developed through examples, reinforced through additional examples, and then applied in a variety of problem-solving situations.

There is a common thread that runs throughout the book:

1. **Learn a skill**
2. **Practice the skill** to help solve equations, and
3. **Apply the skill** to solve application problems

This thread influenced some of the decisions we made in preparing the text.

- When appropriate, problem sets contain an ample number of word problems. Approximately 450 word problems are scattered throughout the text. These problems deal with a variety of applications that show the connection between mathematics and its use in the real world.
- Many problem-solving suggestions are offered throughout the text, and there are special discussions on problem solving in several sections. And when different methods can be used to solve the same problem, those methods are presented for both word problems and other skill problems.
- Newly acquired skills are used as soon as possible to solve equations and inequalities, which, in turn, are used to solve word problems. Therefore, the concept of solving equations and inequalities is introduced early and reinforced throughout the text. The concepts of factoring, solving equations, and solving word problems are tied together in Chapter 3.

In approximately 500 worked-out examples, we demonstrate a wide variety of situations, but we leave some things for students to think about in the problem sets. We also use examples to guide students in organizing their work and to help them decide when they may try a shortcut. The progression from showing all steps to demonstrating a suggested shortcut format is gradual.

As recommended by the American Mathematical Association of Two-Year Colleges, many basic geometry concepts are integrated into a problem-solving setting. This book contains worked-out examples and problems that connect algebra, geometry, and real-world applications. Specific discussions of geometric concepts are contained in the following sections:

Section 2.2 Complementary and supplementary angles; the sum of the measurements of the angles of a triangle equals 180°

Section 2.4 Area and volume formulas

Section 3.4 The Pythagorean theorem

Section 6.2 More on the Pythagorean theorem, including work with isosceles right triangles and 30°–60° right triangles

Content Changes New to This Edition

- Chapter 1 now has a section 0, which reviews fractions. Nearly all students coming into Intermediate Algebra need a review of fractions. Students can assess their current skills in operations with fractions by doing the problems in the Sets of Mastery problems in this section. There are four Sets of Mastery problems in the section. Each set of mastery problems is followed by explanations and examples for students that need remediation on those skills. At the end of the section, there is a problem set with 70 problems.
- Section 2.2 (Equations Involving Fractional Forms) covers solving equations that involve fractions. This section now includes a discussion about distinguishing between an equation and an expression because after learning this section students often misapply the multiplication property of equality to expressions. The Problem Set for the section contains a mixture of equations to solve and expressions to simplify.
- Section 5.2 (Roots and Radicals) material has been reorganized to clarify the presentation of the definitions of roots and their corresponding properties.
- In Chapter 10, the section on solving systems of equations by using matrices and the section on determinants have been removed from the text. The elimination-by-addition method for solving systems of equations has been changed to a more straightforward method.
- A focal point of every revision is the Problem Sets. Some users of the previous edition have suggested that the “very good” Problem Sets could be made even better by adding a few problems in different places. Based on these suggestions, some problems have been added to various problem sets. For example, in Section 10.3 (Elimination-by-Addition Method) many problems were changed to avoid so many fraction answers.

Additional Comments about Some of the Chapters

- Chapter 1 was written so that it can be covered quickly, or on an individual basis if necessary, by those who only need a brief review of some basic arithmetic and algebraic concepts.
- Chapter 2 presents an early introduction to the heart of the intermediate algebra course. Problem solving and the solving of equations and inequalities are introduced early so they can be used as unifying themes throughout the text.
- Chapter 6 is organized to give students the opportunity to learn, on a day-by-day basis, different factoring techniques for solving quadratic equations. The process of completing the square is treated as a viable equation-solving tool for certain types of quadratic equations. The emphasis on completing the square in this setting pays off in Chapter 8 when we graph parabolas, circles, ellipses, and hyperbolas. Section 6.5 offers some guidance as to when to use a particular technique for solving a quadratic equation.
- Chapter 8 was written on the premise that intermediate algebra students should be very familiar with straight lines, parabolas, and circles but have limited exposure to ellipses and hyperbolas.
- In Chapter 9 the definition of a function is built from the definition of a relation. After that, the chapter is devoted entirely to functions; our treatment of the topic does not jump back and forth between functions and relations that are not functions. This chapter includes some work with the composition of functions and the use of linear and quadratic functions in problem-solving situations. In this chapter, domains and ranges are expressed in both interval and set-builder notation. And in the student answer section at the back of the book, domains and ranges are written in both formats.

New Features

Study Skill Tips

These appear at the beginning of each chapter to encourage best study practices throughout the course. A thought-provoking question related to the presented Study Skill Tip encourages students to think more about their current study habits or their past experiences with math.

2

Equations, Inequalities, and Problem Solving

2.1 Solving First-Degree Equations
2.2 Equations Involving Fractional Forms
2.3 Equations Involving Decimals and Problem Solving
2.4 Formulas
2.5 Inequalities
2.6 More on Inequalities and Problem Solving
2.7 Equations and Inequalities Involving Absolute Value

Phil Boorman/Taxi/Getty Images

"The man who thinks he can and the man who thinks he can't are both right."
HENRY FORD

Study Skill Tip

Class time is an intense study time. Start by being prepared physically and mentally for class. For the physical part, consider sitting in the area called the "golden triangle of success." That area is a triangle formed by the front row of the classroom to the middle seat in the back row. This is where the instructor focuses his/her attention. When sitting in the golden triangle of success, you will be apt to pay more attention and be less distracted.

To be mentally prepared for class and note taking, you should practice warming up before class begins. Warming up could involve reviewing the notes from the previous class session, reviewing your homework, preparing questions to ask, trying a few of the unassigned problems, or previewing the section for the upcoming class session. These activities will get you ready to learn during the class session.

Students often wonder if they should be taking notes or just listening. The answer is somewhat different for each student, but every student's notes should contain examples of problems, explanations to accompany those examples, and key rules and vocabulary for the example. The instructor will give clues as to when to write down given information. Definitely take notes when the instructor gives lists such as 1, 2, 3 or A, B, C, says this step is important, or says this problem will be on the test. Through careful listening, you will learn to recognize these clues.

Do you think you can solve word problems?

Chapter Preview

This feature gives a brief description of the material presented in the chapter with student-friendly comments about what to take note of in the chapter.

Apply Your Skill Examples

These present real-life applications so that students can see the relevance of math in everyday life.

Darren Hedges/Shutterstock.com

Classroom Example
It takes a freight train 1 hour longer to travel 180 miles than it takes an express train to travel 195 miles. The rate of the express train is 20 miles per hour greater than the rate of the freight train. Find the times and rates of both trains.

EXAMPLE 6 Apply Your Skill

It takes a freight train 2 hours longer to travel 300 miles than it takes an express train to travel 280 miles. The rate of the express train is 20 miles per hour greater than the rate of the freight train. Find the times and rates of both trains.

Solution

Let t represent the time of the express train. Then $t + 2$ represents the time of the freight train. Let's record the information of this problem in a table.

	Distance	Time	Rate $= \frac{\text{distance}}{\text{time}}$
Express train	280	t	$\frac{280}{t}$
Freight train	300	$t + 2$	$\frac{300}{t+2}$

The fact that the rate of the express train is 20 miles per hour greater than the rate of the freight train can be a guideline.

Rate of express — Equals — Rate of freight train plus 20

$$\frac{280}{t} = \frac{300}{t+2} + 20$$

$$t(t+2)\left(\frac{280}{t}\right) = t(t+2)\left(\frac{300}{t+2} + 20\right) \quad \text{Multiply both sides by } t(t+2)$$

$$280(t+2) = 300t + 20t(t+2)$$

Chapter 4 Summary

OBJECTIVE	SUMMARY	EXAMPLE
Reduce rational numbers and rational expressions. (Section 4.1/Objectives 1 and 2)	Any number that can be written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$, is a rational number. A rational expression is defined as the indicated quotient of two polynomials. The fundamental principle of fractions, $\frac{a \cdot k}{b \cdot k} = \frac{a}{b}$, is used when reducing rational numbers or rational expressions.	Simplify $\frac{x^2 - 2x - 15}{x^2 + x - 6}$. **Solution** $\frac{x^2 - 2x - 15}{x^2 + x - 6} = \frac{(x+3)(x-5)}{(x+3)(x-2)} = \frac{x-5}{x-2}$ **Sample Problem 1** Simplify $\frac{x^2 - 3x - 10}{x^2 + 9x + 14}$.

Sample Problems

Found in the Chapter Summary, a Sample Problem has been added to each Objective to provide students with an opportunity to try a problem similar to the Example presented within the review for each Objective.

Continuing Features

Learning Objectives

Found at the beginning of each section, Learning Objectives are mapped to Problem Sets and to the Chapter Summary.

Examples

More than 700 worked-out Examples show students how to use and apply mathematical concepts. Every Example has a corresponding Classroom Example for the teacher to use.

Explanations

Annotations are in the Examples and provide further explanations of the material.

Classroom Examples

To provide the instructor with more resources, a Classroom Example is written for every Example. Instructors can present these in class or use them for student practice exercises. These Classroom Examples appear in the margin, to the left of the corresponding Example, in both the Annotated Instructor's Edition and in the Student Edition. Answers to the Classroom Examples appear only in the Annotated Instructor's Edition.

Concept Quiz

Every section has a Concept Quiz that immediately precedes the Problem Set. The questions are predominantly true/false questions that allow students to check their understanding of the mathematical concepts and definitions introduced in the section before moving on to their homework. Answers to the Concept Quiz are located at the end of the Problem Set.

Thoughts Into Words

Every Problem Set includes Thoughts Into Words problems, which give students an opportunity to express in written form their thoughts about various mathematical ideas.

Further Investigations

Many Problem Sets include Further Investigations, which allow students to pursue more complicated ideas. Many of these investigations lend themselves to small-group work.

Problem Sets

Problem Sets contain a wide variety of skill-development exercises. Because Problem Sets are a focal point of every revision, problems are added, deleted, and reworded based on users' suggestions.

Chapter Summary

The grid format of the Chapter Summary allows students to review material quickly and easily. Each row of the Chapter Summary includes a Learning Objective, a Summary of that Objective, and a worked-out Example for that Objective with a Sample Problem for students to work.

Chapter Review Problem Sets and Chapter Tests

Chapter Review Problem Sets and Chapter Tests appear at the end of every chapter. Chapter Review Problem Sets give students additional practice, and the Chapter Tests allow students to prepare and practice for "real" tests.

Cumulative Review Problem Sets

Cumulative Review Problem Sets occur about every two chapters. These help students retain skills that were introduced earlier in the text.

Answers

The Answer Section at the back of the text provides answers to the odd-numbered exercises in the Problem Sets and to all problems in the Chapter Review Problem Sets, Chapter Tests, Summary Sample Problems, Cumulative Review Problem Sets, and Appendix A.

Ancillaries

For the Student	For the Instructor
	Annotated Instructor's Edition (ISBN: 978-1-285-19573-5) The Annotated Instructor's Edition provides the complete student text with answers next to each respective exercise, along with answers to the Classroom Examples.
Student Solutions Manual (ISBN: 978-1-285-19701-2) Authors: Karen L. Schwitters, Laurel Fischer The Student Solutions Manual provides worked-out solutions to the odd-numbered problems in the textbook and all solutions for Chapter Reviews, Chapter Tests, and Cumulative Reviews.	**Complete Solutions Manual** (ISBN: 978-1-305-07462-0) Authors: Karen L. Schwitters, Laurel Fischer The Complete Solutions Manual provides worked-out solutions to all of the problems in the textbook.
Student Workbook (ISBN: 978-1-285-19705-0) Author: Maria H. Andersen, former math faculty at Muskegon Community College and now working in the learning software industry The Student Workbook contains the entire student Assessments, Activities, and Worksheets from the Instructor's Resource Binder for classroom discussions, in-class activities, and group work.	**Instructor's Resource Binder** (ISBN: 978-0-538-73675-6) Author: Maria H. Andersen, former math faculty at Muskegon Community College and now working in the learning software industry Each topic in the main text is discussed in uniquely designed Teaching Guides, which contain instruction tips, examples, Activities, Worksheets, overheads, Assessments, and solutions to all Worksheets and Activities.
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For the Student	For the Instructor
	Solution Builder This online database offers complete worked-out solutions to all exercises in the text, allowing you to create customized, secure solutions printouts (in PDF format) matched exactly to the problems you assign in class. For more information, visit www.cengage.com/solutionbuilder.
Math Study Skills Workbook, 4e (ISBN: 978-0-840-05309-1) Author: Paul D. Nolting This best-selling workbook helps traditionally unsuccessful students learn to effectively study mathematics. Typically used for a Math Study Skills course or Freshman Seminar, or as a supplement to class lectures, the Nolting workbook helps students identify their strengths, weaknesses, and personal learning styles in math. Nolting offers proven study tips, test-taking strategies, a homework system, and recommendations for reducing anxiety and improving grades.	**Math Study Skills Workbook, 4e** (ISBN: 978-0-840-05309-1) Author: Paul D. Nolting This best-selling workbook helps traditionally unsuccessful students learn to effectively study mathematics. Typically used for a Math Study Skills course or Freshman Seminar, or as a supplement to class lectures, the Nolting workbook helps students identify their strengths, weaknesses, and personal learning styles in math. Nolting offers proven study tips, test-taking strategies, a homework system, and recommendations for reducing anxiety and improving grades.
Conquering Math Anxiety (with CD-Rom), 3e (ISBN: 978-0-495-82940-9) Author: Cynthia A. Arem This third edition of Arem's Conquering Math Anxiety workbook presents a comprehensive, multifaceted approach to reducing math anxiety and math avoidance.	**Conquering Math Anxiety (with CD-Rom), 3e** (ISBN: 978-0-495-82940-9) Author: Cynthia A. Arem This third edition of Arem's Conquering Math Anxiety workbook presents a comprehensive, multifaceted approach to reducing math anxiety and math avoidance.

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TENTH EDITION

Intermediate Algebra

1 Basic Concepts and Properties

1.0 Review of Fractions

1.1 Sets, Real Numbers, and Numerical Expressions

1.2 Operations with Real Numbers

1.3 Properties of Real Numbers and the Use of Exponents

1.4 Algebraic Expressions

Paul Bradbury/OJO Images/Getty Images

"Before beginning, prepare carefully"
MARCUS TULLIUS CICERO, ANCIENT ROMAN SCHOLAR

Study Skill Tip

There are many factors that affect success in a math course, such as the instructor, the textbook, your motivation, time of the day for class, etc. However, one of the most important factors for success is being placed in the right course for you. Can you imagine taking French II without having taken French I? What would be the likelihood of being successful? If at the beginning of this course, you think the material is way too difficult or way too easy, talk to your instructor regarding your placement in this course.

Two other factors that are extremely important for success in a math course are attending class regularly and doing the homework. If at all possible, don't ever miss class. However, take action right now and find a classmate whom you can contact in case you miss class. Get the names and college email addresses of several fellow students whom you could possibly contact to get the class notes in case you miss class.

Also, know the resources available if you need help with the homework. Become aware of your instructor's office hours and the location of any tutoring centers on campus. Also consider utilizing websites for additional help with your math course. Your instructor or fellow classmates can usually suggest appropriate websites for Intermediate Algebra.

Are you prepared enough to feel confident about your success in this algebra class?

Chapter Preview

An Intermediate Algebra course assumes that you have basic arithmetic skills, including fractions and basic algebra skills. This chapter includes a review of fractions in Section 1.0. The section is written with diagnostic problems to help you determine whether you have mastery of the basic operations with fractions. I encourage you to try the four sets of mastery problems in Section 1.0 even if your instructor does not assign Section 1.0. The answers to the mastery sets are in the back of the book.

Algebra is often described as *generalized arithmetic*. That description does convey an important idea: A good understanding of arithmetic provides a sound basis for the study of algebra. In this chapter we use the concepts of *numerical expression* and *algebraic expression* to review some ideas from arithmetic and begin the transition to algebra. Be sure you thoroughly understand the basic concepts reviewed in this first chapter.

1.0 Review of Fractions

As with any math course, you need the prerequisite skills in order to be successful with the new material presented. Students enter into Intermediate Algebra with varying levels of math proficiency. Some students have a strong background and come into the course fully prepared. Other students may have a weak background or have not been enrolled in a math course for a while and have forgotten the prerequisite skills.

Throughout this section, problems will be presented for you to determine your mastery of some of the prerequisite arithmetic and algebra skills. There will be ten problems. Answers to these problems are in the back of the book. Use the following legend as a guide to direct your studying of the material immediately following the problems.

Number of problems correct	Prescription
10 or 9	You probably have mastery of this skill and can go on to the next topic without reviewing.
8 or 7	You have some basic mastery of this skill but do need to review. Read the material and do the corresponding problems in the problem set to gain mastery of this skill.
6 or less	You have not mastered this skill. Read the material and do the corresponding problems in the problem set to gain mastery of this skill. If after that you are still not proficient in this skill, ask your instructor for additional study materials.

Try the following problems to help determine your proficiency with prime numbers.

Mastery Set 1 Prime Numbers

For Problems 1–4, label the number as prime or composite.

1. 9 **2.** 30 **3.** 41 **4.** 57

For Problems 5–7, factor the composite number into a product of prime numbers.

5. 40 **6.** 84 **7.** 210

For Problems 8–10, find the least common multiple of the given numbers.

8. 4 and 18 **9.** 6, 18, and 21 **10.** 4, 10, and 15

Prime Numbers

Because prime numbers and prime factorization play an important role in the operations with fractions, let's begin by considering two special kinds of whole numbers: prime numbers and composite numbers.

Definition 1.1

A **prime number** is a whole number greater than 1 that has no factors (divisors) other than itself and 1. Whole numbers greater than 1 that are not prime numbers are called **composite numbers**.

The prime numbers less than 50 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, and 47. Note that each of these has no factors other than itself and 1. We can express every composite number as the indicated product of prime numbers. Consider the following examples:

$$4 = 2 \cdot 2 \qquad 6 = 2 \cdot 3 \qquad 8 = 2 \cdot 2 \cdot 2 \qquad 10 = 2 \cdot 5 \qquad 12 = 2 \cdot 2 \cdot 3$$

In each case we express a composite number as the indicated product of prime numbers. This form is called the prime-factored form of the number. There are various procedures to find the prime factors of a given composite number. For our purposes, the simplest technique is to factor the given composite number into any two easily recognized factors and then continue to factor each of these until we obtain only prime factors. Consider these examples:

$$18 = 2 \cdot 9 = 2 \cdot 3 \cdot 3 \qquad 27 = 3 \cdot 9 = 3 \cdot 3 \cdot 3$$
$$24 = 4 \cdot 6 = 2 \cdot 2 \cdot 2 \cdot 3 \qquad 150 = 10 \cdot 15 = 2 \cdot 5 \cdot 3 \cdot 5$$

It does not matter which two factors we choose first. For example, we might start by expressing 18 as $3 \cdot 6$ and then factor 6 into $2 \cdot 3$, which produces a final result of $18 = 3 \cdot 2 \cdot 3$. Either way, 18 contains two prime factors of 3 and one prime factor of 2. The order in which we write the prime factors is not important.

Least Common Multiple

It is sometimes necessary to determine the smallest common nonzero multiple of two or more whole numbers. We call this nonzero number the **least common multiple**. In our work with fractions, there will be problems for which it will be necessary to find the least common multiple of some numbers—usually the denominators of fractions. So let's review the concepts of multiples. The set of all whole numbers that are multiples of 5 consists of 0, 5, 10, 15, 20, 25, and so on. In other words, 5 times each successive whole number ($5 \cdot 0 = 0$, $5 \cdot 1 = 5$, $5 \cdot 2 = 10$, $5 \cdot 3 = 15$, and so on) produces the multiples of 5. In a like manner, the set of multiples of 4 consists of 0, 4, 8, 12, 16, and so on. We can find the least common multiple of 5 and 4 by using a simple listing of the multiples of 5 and the multiples of 4.

Multiples of 5 are 0, 5, 10, 15, 20, 25, 30, 35, 40, 45, . . .

Multiples of 4 are 0, 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, . . .

The nonzero numbers in common on the lists are 20 and 40. The least of these, 20, is the least common multiple. Stated another way, 20 is the smallest nonzero whole number that is divisible by both 4 and 5.

From your knowledge of arithmetic, you will often be able to determine the least common multiple by inspection. For instance, the least common multiple of 6 and 8 is 24.

Therefore, 24 is the smallest nonzero whole number that is divisible by both 6 and 8. If we cannot determine the least common multiple by inspection, then using the prime-factored form of composite numbers is helpful. The procedure is as follows.

Step 1 Express each number as a product of prime factors.

Step 2 The least common multiple contains each different prime factor. For each different factor, determine the most times each different factor is used in any of the factorizations. Those factors will then be used that number of times in the least common multiple. (For example, if the factor 2 occurs at most three times in any factorization, then the least common multiple will have the factor 2 used three times.)

The following examples illustrate this technique for finding the least common multiple of two or more numbers.

Classroom Example
Find the least common multiple of 8 and 30.

EXAMPLE 1 Find the least common multiple of 24 and 36.

Solution

Let's first express each number as a product of prime factors.

$$24 = 2 \cdot 2 \cdot 2 \cdot 3$$
$$36 = 2 \cdot 2 \cdot 3 \cdot 3$$

There are two different factors, 2 and 3, in the prime-factored forms.

The prime factor 2 occurs the most times (three times) in the factorization of 24. Because the factorization of 24 contains three 2s, the least common multiple must have three 2s.

The prime factor 3 occurs the most times (two times) in the factorization of 36. Because the factorization of 36 contains two 3s, the least common multiple must have two 3s.

The least common multiple of 24 and 36 is therefore $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 = 72$.

Classroom Example
Find the least common multiple of 42 and 60.

EXAMPLE 2 Find the least common multiple of 48 and 84.

Solution

$$48 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$$
$$84 = 2 \cdot 2 \cdot 3 \cdot 7$$

There are three different factors, 2, 3, and 7, in the prime-factored forms.

The most number of times that 2 occurs is four times in the factored form of 48.

The factors 3 and 7 only occur once in each factored form, so we need one factor of each for the least common multiple.

The least common multiple of 48 and 84 is $2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 7 = 336$.

Classroom Example
Find the least common multiple of 10, 15, and 24.

EXAMPLE 3 Find the least common multiple of 12, 18, and 28.

Solution

$$28 = 2 \cdot 2 \cdot 7$$
$$18 = 2 \cdot 3 \cdot 3$$
$$12 = 2 \cdot 2 \cdot 3$$

There are three different factors, 2, 3, and 7, in the prime-factored forms.

The most number of times that 2 occurs in any factored form is twice, so we need two factors of 2 in the least common multiple.

The most number of times that 3 occurs in any factored form is twice, so we need two factors of 3 in the least common multiple.

The factor, 7, only occurs once in the factored forms, so we need one factor of 7 for the least common multiple.

The least common multiple is $2 \cdot 2 \cdot 3 \cdot 3 \cdot 7 = 252$.

Classroom Example
Find the least common multiple of 6 and 25.

EXAMPLE 4 Find the least common multiple of 8 and 9.

Solution

$$9 = 3 \cdot 3$$
$$8 = 2 \cdot 2 \cdot 2$$

There are two different factors, 2 and 3, in the prime-factored forms.

The most number of times that 2 occurs in any factored form is three times, so we need three factors of 2 in the least common multiple.

The most number of times that 3 occurs in any factored form is twice, so we need two factors of 3 in the least common multiple.

The least common multiple is $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 = 72$.

Try the following problems to help determine your proficiency with reducing, multiplying, and dividing fractions.

Mastery Set 2 Reducing, Multiplying, and Dividing Fractions

For Problems 1–3, reduce the fraction to lowest terms.

1. $\frac{10}{25}$ **2.** $\frac{12}{34}$ **3.** $\frac{14}{42}$

For Problems 4–7, multiply the fractions and express the answer in lowest terms.

4. $\frac{1}{5} \cdot \frac{2}{7}$ **5.** $\frac{4}{3} \cdot \frac{9}{24}$ **6.** $\frac{12}{5} \cdot \frac{1}{9}$ **7.** $\frac{7}{2} \cdot \frac{12}{7}$

For Problems 8–10, divide the fractions and express the answer in lowest terms.

8. $\frac{6}{5} \div \frac{2}{3}$ **9.** $\frac{21}{5} \div \frac{14}{15}$ **10.** $4 \div \frac{1}{5}$

Reducing Fractions

Before we proceed too far with operations on fractions, we need to learn about reducing fractions. The following property is applied throughout our work with fractions. We call this property the fundamental property of fractions.

Fundamental Property of Fractions

If b and k are nonzero integers, and a is any integer, then $\frac{a \cdot k}{b \cdot k} = \frac{a}{b}$.

The fundamental property of fractions provides the basis for what is often called reducing fractions to lowest terms, or expressing fractions in simplest or reduced form. Let's apply the property to a few examples.

Classroom Example
Reduce $\frac{25}{35}$ to lowest terms.

EXAMPLE 5 Reduce $\frac{12}{18}$ to lowest terms.

Solution

$$\frac{12}{18} = \frac{2 \cdot \cancel{6}}{3 \cdot \cancel{6}} = \frac{2}{3}$$

A common factor of 6 has been divided out of both numerator and denominator

Classroom Example
Change $\frac{18}{50}$ to simplest form.

EXAMPLE 6 Change $\frac{14}{35}$ to simplest form.

Solution

$$\frac{14}{35} = \frac{2 \cdot \cancel{7}}{5 \cdot \cancel{7}} = \frac{2}{5}$$

A common factor of 7 has been divided out of both numerator and denominator

Classroom Example
Reduce $\frac{24}{28}$.

EXAMPLE 7 Reduce $\frac{72}{90}$.

Solution

$$\frac{72}{90} = \frac{\cancel{2} \cdot 2 \cdot 2 \cdot \cancel{3} \cdot \cancel{3}}{\cancel{2} \cdot \cancel{3} \cdot \cancel{3} \cdot 5} = \frac{4}{5}$$

The prime-factored forms of the numerator and denominator may be used to find common factors

Multiplying Fractions

We are now ready to consider multiplication problems with the understanding that the final answer should be expressed in reduced form. Study the following examples carefully; we use different methods to simplify the problems.

We can define the multiplication of fractions in common fractional form as follows.

Multiplying Fractions

If a, b, c, and d are integers, with b and d not equal to zero, then $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$.

To multiply fractions in common fractional form, we simply multiply numerators and multiply denominators. The following examples illustrate the multiplying of fractions.

$$\frac{1}{3} \cdot \frac{2}{5} = \frac{1 \cdot 2}{3 \cdot 5} = \frac{2}{15}$$

$$\frac{3}{4} \cdot \frac{5}{7} = \frac{3 \cdot 5}{4 \cdot 7} = \frac{15}{28}$$

$$\frac{3}{5} \cdot \frac{5}{3} = \frac{15}{15} = 1$$

The last of these examples is a very special case. If the product of two numbers is 1, then the numbers are said to be reciprocals of each other.

Classroom Example

Multiply $\left(\frac{18}{5}\right)\left(\frac{15}{14}\right)$.

EXAMPLE 8 Multiply $\left(\frac{9}{4}\right)\left(\frac{14}{15}\right)$.

Solution

$$\left(\frac{9}{4}\right)\left(\frac{14}{15}\right) = \frac{\cancel{3} \cdot 3 \cdot \cancel{2} \cdot 7}{\cancel{2} \cdot 2 \cdot \cancel{3} \cdot 5} = \frac{21}{10}$$ Factor each numerator and denominator and then reduce

Classroom Example

Find the product of $\frac{4}{7}$ and $\frac{21}{8}$.

EXAMPLE 9 Find the product of $\frac{8}{9}$ and $\frac{18}{24}$.

Solution

$$\frac{\overset{1}{\cancel{8}}}{\underset{1}{\cancel{9}}} \cdot \frac{\overset{2}{\cancel{18}}}{\underset{3}{\cancel{24}}} = \frac{2}{3}$$ A common factor of 8 has been divided out of 8 and 24, and a common factor of 9 has been divided out of 9 and 18

Dividing Fractions

The next example motivates a definition for division of rational numbers in fractional form:

$$\frac{\frac{3}{4}}{\frac{2}{3}} = \left(\frac{\frac{3}{4}}{\frac{2}{3}}\right)\left(\frac{\frac{3}{2}}{\frac{3}{2}}\right) = \frac{\left(\frac{3}{4}\right)\left(\frac{3}{2}\right)}{1} = \left(\frac{3}{4}\right)\left(\frac{3}{2}\right) = \frac{9}{8}$$

Note that $\left(\frac{\frac{3}{2}}{\frac{3}{2}}\right)$ is a form of 1, and $\frac{3}{2}$ is the reciprocal of $\frac{2}{3}$. In other words, $\frac{3}{4}$ divided by $\frac{2}{3}$ is equivalent to $\frac{3}{4}$ times $\frac{3}{2}$. The following definition for division now should seem reasonable.

> **Division of Fractions**
>
> If b, c, and d are nonzero integers, and a is any integer, then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$.

Note that to divide $\frac{a}{b}$ by $\frac{c}{d}$, we multiply $\frac{a}{b}$ times the reciprocal of $\frac{c}{d}$, which is $\frac{d}{c}$. The next examples demonstrate the important steps of a division problem.

$$\frac{2}{3} \div \frac{1}{2} = \frac{2}{3} \cdot \frac{2}{1} = \frac{4}{3}$$

$$\frac{5}{6} \div \frac{3}{4} = \frac{5}{6} \cdot \frac{4}{3} = \frac{5 \cdot 4}{6 \cdot 3} = \frac{5 \cdot \cancel{2} \cdot 2}{\cancel{2} \cdot 3 \cdot 3} = \frac{10}{9}$$

$$\frac{6}{7} \div 2 = \frac{6}{7} \div \frac{2}{1} = \frac{\overset{3}{\cancel{6}}}{7} \cdot \frac{1}{\underset{1}{\cancel{2}}} = \frac{3}{7}$$

Classroom Example
Divide $\frac{2}{5} \div \frac{2}{3}$.

EXAMPLE 10 Divide $\frac{4}{9} \div \frac{3}{2}$.

Solution

$\frac{4}{9} \div \frac{3}{2} = \frac{4}{9} \cdot \frac{2}{3} = \frac{8}{27}$ Multiply by the reciprocal of $\frac{3}{2}$

Classroom Example
Divide $\frac{21}{4} \div 3$.

EXAMPLE 11 Divide $\frac{7}{2} \div 3$.

Solution

$\frac{7}{2} \div 3 = \frac{7}{2} \div \frac{3}{1}$ Rewrite 3 as $\frac{3}{1}$

$= \frac{7}{2} \cdot \frac{1}{3} = \frac{7}{6}$ Multiply by $\frac{1}{3}$, the reciprocal of $\frac{3}{1}$

Try the following problems to help determine your proficiency with addition and subtraction of fractions.

Mastery Set 3 Adding and Subtracting Fractions

For Problems 1–5, perform the addition. Express the answer in lowest terms.

1. $\frac{1}{8} + \frac{5}{8}$ **2.** $\frac{3}{4} + \frac{1}{8}$ **3.** $\frac{3}{5} + \frac{2}{3}$ **4.** $\frac{7}{12} + \frac{3}{8}$ **5.** $\frac{11}{60} + \frac{13}{24}$

For Problems 6–9, perform the subtraction. Express the answer in lowest terms.

6. $\frac{6}{7} - \frac{2}{7}$ **7.** $\frac{11}{12} - \frac{1}{4}$ **8.** $\frac{7}{8} - \frac{4}{5}$ **9.** $\frac{7}{18} - \frac{5}{24}$

10. If Jessica ate $\frac{3}{8}$ of a pepperoni pizza and $\frac{1}{4}$ of a cheese pizza, what was her total portion of pizza eaten?

Adding and Subtracting Fractions

Suppose that it is one-fifth of a mile between your dorm and the union and two-fifths of a mile between the union and the library along a straight line, as indicated in Figure 1.1. The total distance between your dorm and the library is three-fifths of a mile, and we write $\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$.

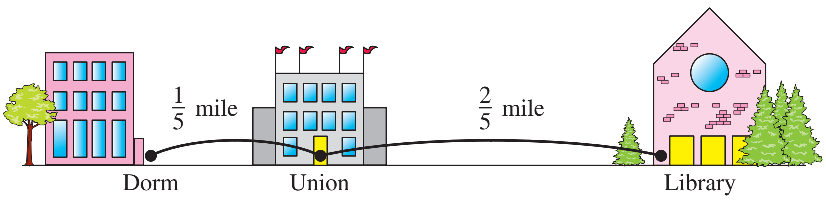

Figure 1.1

A pizza is cut into seven equal pieces and you eat two of the pieces (see Figure 1.2). How much of the pizza remains? We represent the whole pizza by $\frac{7}{7}$ and conclude that $\frac{7}{7} - \frac{2}{7} = \frac{5}{7}$ of the pizza remains.

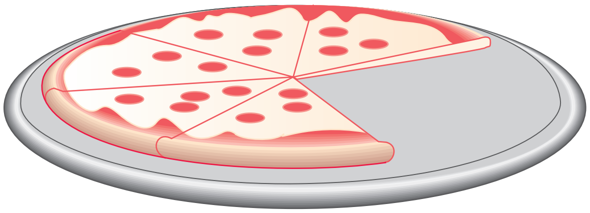

Figure 1.2

These examples motivate the following definition for addition and subtraction of rational numbers in $\frac{a}{b}$ form.

Addition and Subtraction of Fractions

If a, b, and c are integers, and b is not zero, then

$$\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b} \qquad \text{Addition}$$

$$\frac{a}{b} - \frac{c}{b} = \frac{a - c}{b} \qquad \text{Subtraction}$$

We say that fractions with common denominators can be added or subtracted by adding or subtracting the numerators and placing the results over the common denominator. Consider the following examples:

$$\frac{3}{7} + \frac{2}{7} = \frac{3 + 2}{7} = \frac{5}{7}$$

$$\frac{7}{8} - \frac{2}{8} = \frac{7 - 2}{8} = \frac{5}{8}$$

$$\frac{5}{6} - \frac{1}{6} = \frac{5 - 1}{6} = \frac{4}{6} = \frac{2}{3}$$ We agree to reduce the final answer

How do we add or subtract if the fractions do not have a common denominator? We use the fundamental principle of fractions, $\frac{a \cdot k}{b \cdot k} = \frac{a}{b}$, to get equivalent fractions that have a common denominator. **Equivalent fractions** are fractions that name the same number. Consider the next example, which shows the details.

Classroom Example

Add $\frac{2}{3} + \frac{4}{7}$.

EXAMPLE 12 Add $\frac{1}{4} + \frac{2}{5}$.

Solution

$$\frac{1}{4} = \frac{1 \cdot 5}{4 \cdot 5} = \frac{5}{20}$$ $\frac{1}{4}$ and $\frac{5}{20}$ are equivalent fractions

$$\frac{2}{5} = \frac{2 \cdot 4}{5 \cdot 4} = \frac{8}{20}$$ $\frac{2}{5}$ and $\frac{8}{20}$ are equivalent fractions

$$\frac{5}{20} + \frac{8}{20} = \frac{13}{20}$$

Note that in Example 12 we chose 20 as the common denominator, and 20 is the least common multiple of the original denominators 4 and 5. (Recall that the least common